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A Machine Breakdown Model with Fuzzy Environment Implication and Volume Flexibility

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Abstract—*Production, quality and maintenance are the three major concerns of any manufacturing firm. In the competitive business environment, managers of manufacturing industries encounter the challenge every day to produce quality products and to provide better services than before to customers. Due to technological innovations and scientific developments around the world, manufacturing infrastructure is also changing rapidly. Three important factors of the EMQ (Economic Manufacturing Quantity) model have been dealt with prior significance. In the present study, we developed a production model with the uniform distributed deterioration and machine breakdown underfuzzy environment. As a result the total cost function is ultimately obtained as fuzzy. Later on this cost function is defuzzified to obtain a crisp cost function with allowed variations. This is solved and analyzed theoretically and numerically.*

Keywords*: Uniform Distribution Deterioration, Machine breakdown, fuzzification.*

9.1 Introduction

Production, quality and maintenance are the three major concerns of any manufacturing firm. In the competitive business environment, managers of manufacturing industries encounter the challenge everyday to produce quality products and to provide better services than before to customers. Due to technological innovations and scientific developments around the world, manufacturing infrastructure is also changing rapidly. Three important factors of the Economic Manufacturing Quantity (EMQ) model have been dealt with prior significance. First it is assumed that the production facility is not perfect reliable. Second, the production rate (greater than the demand rate) is not predetermined and fixed in advance. Third, the modern facilities are not free from deterioration due to epoch. As a result, random machine shifts from 'in-control' state to 'out-of-control' state frequently occur during production runs and some percentage of non-conforming items is produced. Further, the process deterioration after a machine shift may result in a machine breakdown in which case the interrupted lot is usually aborted and then the basic EMQ model loses its usefulness. So, from theoretical as well as practical view points, the study of EMQ problem for unreliable manufacturing systems is quite significant and meaningful. There a major vacant space in the area of inventory modeling with machine breakdown. So, we have also taken a stride at the forefront to solve out the machine breakdown problem with flexible manufacturing system.

Chung (1997) discussed about the bound of machine breakdown problem. **Giri,** and **Dohi, (2005)** presented an exact formulation of EMQ model under a general framework in which the time to machine failure, corrective and preventive repair times all are assumed to follow arbitrary probability distributions. But they ignored the study of fuzzy**. Chakraborty** *etal.* **(2008)** presents a generalized economic manufacturing quantity model for an unreliable production system in which the production facility may shift from an 'in-control' state to an 'out-of-control' state at any random time and may ultimately break down afterwards. **Singh** and **Urvashi (2010)** discussed the effect of machine breakdown with fuzzy demand rate. They considered the volume flexibility with idle time of the inventory system**. Widyadana** and **Wee (2010)** develops deteriorating items production inventory models with random machine breakdown and stochastic repair time. The model assumes the machine repair time is independent of the machine breakdown rate.

Certainty eventually indicates that we assume the structures and parameters of the model to be definitely known, and that there are no doubts about their values or occurrence. In reality it is not possible, so fuzzificztion of parameters is considered in system. **Vojosevic***et al.***(1996)** and **Chen** and **Wang (1996)**fuzzified the ordering cost into trapezoidal fuzzy number in the total cost of an inventory without backorder model and obtained the fuzzy total cost. **Chang** *et al.* **(2006)** considered the mixture inventory model involving a fuzzy random variable and obtained the total cost in the fuzzy sense. **Rong***et al.* **(2008)** presented an optimization inventory policy for a deteriorating item with partially/fully backlogged shortages and price dependent demand. **Singh, S.R.** and **Singh, C. (2008)** considered the fuzzy inventory model for finite rate of replenishment using signed distance method. **Singh S.R.** and **Urvashi (2010)** also discussed on fuzzy inventory model with different conditions. In this paper manufacturing system considered with capacity constraints.

In conventional studies of inventory models, it is normally assumed that the lifetime of an item is infinite while in storage. In reality, it is not always true. Due to the unsatisfactory preservation conditions, some portions of the items like food grains, vegetables, fruits, radioactive substances, fashion goods, blood, high- tech products, drugs, etc., are damaged or decayed due to spoilage, obsolescence, evaporation, pilferage, etc. and are not in a condition to satisfy the customer's demand. **Nahmias (1982)** classified the deteriorating inventory problems into two categories: fixed lifetime and random lifetime. Many researchers, like **Mandal** and **Phaujder (1989), Ting** and **Chung (1994), Hariga (1994), Mandal** and **Maiti (1999)**, etc. considered the inventory models for deteriorating items assuming fixed life cycle. **Roy** *et al.* **(2010)** develops an inventory model of a volume flexible manufacturing system for a deteriorating item with randomly distributed shelf life, continuous time-varying demand, and shortages over a finite time horizon. Total cost is derived for the system and minimized.

Many of the researchers have considered single item inventory models with crisp parameter only. In the past, researchers pay less attention towards the coordination of the factor of machine breakdown, volume flexibility and fuzzy environment with multi items which proves a major hindrance to a researcher in this field. It is very much realistic condition for business environment. Produced units deteriorate over time. But most of researchers consider certainty in deterioration. In reality items deteriorate with uncertainty that follows different distributions. Therefore, we have developed this entire concept simultaneously in our model with uniform distribution of deterioration.

This chapter investigated an Economic Manufacturing Quantity model for time dependent decaying items and selling price demand with volume flexible environment. Controlling market demand through the manipulation of selling price is an important strategy for increasing profit. We assumed that different machines 'A_i' (1,2….n) are dedicated to the production of different items 'i' with different production rates 'P_i'. The management of production in machine A_i is vested with the management unit 'B_i'. It is assumed that a machine may become out of order during its working time. As a result, there is a mean time for every machine between its failure/breakdowns. During a breakdown of a machine, there is demand although there is no production. In such a situation, the demand is met until the inventory level falls below the quantity demanded. When inventory level becomes less than the demand, the concerned management unit B_i is rendered fully idle. This situation occurs when the customer is a wholesaler having the demand of a big lot size and the concerned management unit can't meet this demand because the stock size is less than the quantity demanded. Therefore, we considered the idle time of each management unit; this idle time leads to an additional cost for the last man hours. We have considered the capital available for manufacturing the items is limited. In the present study, demand of multi items involved in the study is represented by fuzzy numbers. As a result the total cost function is ultimately obtained as fuzzy. Later on this cost function is defuzzified to obtain a crisp cost function with allowed variations. This chapter is alienated into two parts: (1) first one with constant deterioration and selling price dependent demand and (2) second one with the uniform distributed deterioration and time dependent demand. This is solved and analyzed theoretically.

9.2 Assumptions and Nomenclature: The proposed inventory model is developed under the following assumptions and notations:

9.2.1 Assumptions:

The following assumptions are made for development of mathematical model:

- Model is developed for multiple items.
- Demand rate is selling price dependent for each item for first model and time dependent for the second one model.
- Production rate is considered as a decision variable.
- Machine breakdown is considered during the production period.
- Idle time is considered for management of units.
- Crisp and fuzzy both the cases are considered.

9.2.2 Nomenclature:

The following notations are made for development of mathematical model:

$Q_i(t)$: On- hand inventory of i-th item at time t

P_i: Production rate per unit time for the i-th item

$\theta(t)$: Deterioration rate

μ_i: Mean time between successive breakdown of the machine

m_i: Mean time of repair of i-th machine

τ_i: Mean duration of a breakdown of machine

$\psi_i(t_i)$: Probability density function of t_i

$\phi_i(\tau_i)$: Probability density function of τ_i

C_h^i: Cost of carrying one unit of i-th item in inventory per unit time

C_s^i: Shortage Costper unit time of the i-th item

S_p^i: Selling price per unit of i-th item

$\eta_i(P_i)$: Cost for production of a unit of i-th item

D_i: Demand rate for the i-th item per unit time

W_i: Cost of idle time of management unit Bi

CAP: Total capital available for production of all the items

9.3 Model I: Formulation of the Model with Selling Price Dependent Demand:

The production cycle begins with zero stock. Production starts at time t=0 and stocks reaches at the highest level $Q_i(t_i)$ at time t_i. After time t_i machine becomes out of order, then repairing of machines starts and takes time to come back into working state. During the repairing period two cases may arise: one is scenario 9 (a) which is very simple and unrealistic case second is scenario 9 (b) which is very common in manufacturing firms. Hence, our main object is to analyze scenario 1 (b).

Differential Equations of the inventory system are

$$Q_i^{'}(t)+\theta Q_i(t)=P_i-D_i(s) \quad \text{.... (9.3.1)} \quad \text{with } Q_i(0)=0$$

$$Q_i^{'}(t)+\theta Q_i(t)=-D_i(s) \quad \text{....(9.3.2) with } Q_i(x)=0$$

Solutions of the above equations are

$$Q_i(t)=\frac{(P_i-D_i(s))}{\theta}(1-e^{-\theta t}) \quad \text{.... (9.3.3)}$$

$$Q_i(t)=-\frac{D_i(s)}{\theta}(1-e^{\theta(x-t)}) \quad \text{.... (9.3.4)}$$

Scenario 9(a):

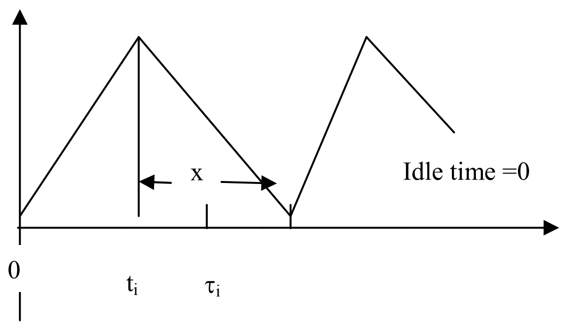

Figure 9.1: Graphical representation of inventory system without shortages

Scenario 9 (b):

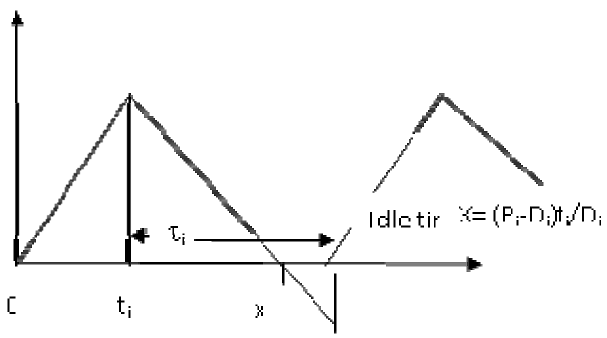

Figure 9.2: Graphical representation of inventory system with shortages

One can conclude that the idle times of the management units {Bi, i= 1, 2, 3…..}due to breakdown of the machines {Ai, i=1, 2,3…..} are

$$u_i = \begin{cases} 0, & \text{if } \dfrac{Q_i(t_i)}{D_i(s)} \geq \tau_i \\ \tau_i-\dfrac{Q_i(t_i)}{D_i(s)} & \text{if } \dfrac{Q_i(t_i)}{D_i(s)} < \tau_i \end{cases}$$

9.3.1 Expected cost per breakdown

The expected cost per breakdown of the machine {Ai, i= 1, 2, 3……} during idle time, is

$$E_{ic}{}^{i}=W_i\int_0^{\infty}\{\int_{\frac{Q_i(t_i)}{D_i(s)}}^{\infty}(\tau_i-\frac{Q_i(t_i)}{D_i(s)})\phi_i(\tau_i)d\tau_i\}\psi_i(t_i)dt_i \quad \text{.... (9.3.5)}$$

9.3.2 Expected shortage cost during ideal time

The expected shortage cost for i-th item, during idle item, is

$$E_{sc}{}^{i}=C_S{}^{i}D_i(s)\int_0^{\infty}\{\int_{\frac{Q_i(t_i)}{D_i(s)}}^{\infty}(\tau_i-\frac{Q_i(t_i)}{D_i(s)})\phi_i(\tau_i)d\tau_i\}\psi_i(t_i)dt_i \quad \text{.... (9.3.6)}$$

Now the total inventory of i-th item is

$Inv_i(t_i)$ = Inventory during$[0, t_i]$ + Inventory during [0, x]

$$=\frac{(P_i-D_i(s))}{\theta}\int_0^{t_i}(1-e^{-\theta t})dt-\frac{D_i(s)}{\theta}\int_0^{x}(1-e^{\theta(x-t)})dt \quad \text{.... (9.3.7)}$$

9.3.3 Expected inventory cost: Expecting Holding cost for i-th item

$$E_{ic}{}^{i}=C_h^i\int_0^{\infty}Inv_i(t_i)\psi_i(t_i)dt_i \quad \text{.... (9.3.8)}$$

9.3.4 Production cost: Production cost per unit of the item for the given system is

$$\eta_i(P_i)=(R_i+\frac{G_i}{P_i^{\alpha}}+k_iP_i^{\gamma}) \quad \text{.... (9.3.9)}$$

Here one can consider the density functions are

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$$\psi_i(t_i) = \frac{1}{\mu_i} e^{-t_i/\mu_i}, \ \phi_i(\tau_i) = \frac{1}{m_i} e^{-\tau_i/m_i}$$

9.3.5 Expected total cost

Expected total cost breakdown, including the inventory and shortages cost is,

ETC (P_1, P_2, P_3, ……) = Expected holding cost + Expected cost for idle item + Expected shortage cost

ETC (P_1, P_2, P_3……) = $=f(P_i)D_i+g(P_i)$ …. (9.3.11)

Where

$$f(P_i) = \sum_{i=1}^{n} \frac{(C_s^i D_i(s)+W_i)m_i^2}{\mu_i(P_i-D_i(s))+m_i D_i(s)} + \sum_{i=1}^{n} C_h^i \left[\frac{P_i-D_i(s)}{D_i(s)}\left(\mu_i + \frac{\mu_i}{\theta} + 1 - \frac{1}{\theta} - \frac{\mu_i D_i(s)}{\theta}\right) - \frac{1}{\theta}\left(\frac{D_i(s)}{\theta(D_i(s)-\theta\mu_i(P_i-D_i(s)))} - \frac{1}{\theta}\right)\right]$$

And $g(P_i) = \sum_{i=1}^{n} \eta_i(P_i)\, P_i \mu_i$

9.3.6 Expected Production cost of the item

$E_{prc} = \sum_{i=1}^{n} \int_0^{\infty} \eta_i(P_i) P_i t_i \ \psi_i(t_i) dt_i$ …. (9.3.12)

As the capital for manufacturing the item is limited, the constraints

$\sum_{i=1}^{n} \eta_i(P_i) P_i \mu_i \le CAP$ must be satisfied.

9.3.7 Mathematical formulation of the fuzzy model:

When the demand rate becomes fuzzy, the objective function can be redefined as

ETC (P_1, P_2, P_3……) = $= f(P_i)\tilde{D}_i(s) + g(P_i)$

Wavy bar denotes the fuzzification of the parameters. We express the fuzzy demand rate $\widetilde{D_i}$ as the triangular fuzzy number $(D_i(s)-\delta_1, D_i(s), D_i(s)+\delta_2)$. Suppose, the membership function of the fuzzy demand rate $\widetilde{D_i}$ is as follows:

$$\mu_{\tilde{D}_i}(D_i) = \begin{cases} \dfrac{D - D_i + \delta_1}{\delta_1} & D_i - \delta_1 \le D \le D_i \\ \dfrac{D_i + \delta_2 - D}{\delta_2} & D_i \le D \le D_i + \delta_2 \\ 0 & elsewhere \end{cases}$$

Here, 0< δ_1 <D_i, 0< δ_2 and D_i are given fixed numbers δ_1 and δ_2 are determined by the decision maker based on the given uncertainty. From equation (11), for each P_i , let

$$G_{P_i}(D_i) = f(P_i)D_i + g(P_i)$$

and y= $G_{P_i}(D_i)$ then we have $D_i = \dfrac{G_{P_i}(D_i) - g(P_i)}{f(P_i)}$

By the extension Principle, one can have the following

$$\mu_{G_{P_1}(D_i)}(y) = \sup_{D_i \in G_{P_1}(D_i)} \mu_{\tilde{D}_i}(D_i) = \mu_{\tilde{D}_i}\left(\frac{G_{P_i}(D_i) - g(P_i)}{f(P_i)}\right)$$

$$= \begin{cases} \dfrac{G_{P_i}(D_i) - g(P_i) - (D_i - \delta_1) f(P_i)}{f(P_i)\delta_1} & (D_i - \delta_1) f(P_i) + g(P_i) \le G_{P_i}(D_i) \le D_i f(P_i) + g(P_i) \\ \dfrac{(D_i + \delta_2) f(P_i) - G_{P_i}(D_i) + g(P_i)}{f(P_i)\delta_2} & D_i f(P_i) + g(P_i) \le G_{P_i}(D_i) \le (D_i + \delta_2) f(P_i) + g(P_i) \\ 0 & elsewhere \end{cases}$$

The centroid of $\mu_{G_{P_1}(D_i)}(y)$ is

M(P_i, δ_1,δ_2) =R/P= $D_0 f(P_i) - g(P_i) - \dfrac{f(P_i)(\delta_1 - \delta_2)}{3}$ …. (9.3.13)

where, 0< $\delta_1 < D_0$, 0< δ_2 .

M(P_i, δ_1,δ_2) is the estimate of total cost in fuzzy sense.

If $\delta_1 = \delta_2$, then equation (9.11) reduces to (9.12).

Equation (9.13) gives an estimate of cost function. For minimization of cost function, differentiating equations (9.13) with respect to P_1, P_2. The software **Mathematica 8.0** is used to derive the optimal solution.

9.4 Numerical Illustration

The model has been explored numerically as well. We have considered the following data for the study which based on the previous study. The common input parameters are:

Items no.(i)	W_i	μ_i	m_i	R_i	k_i	G_i	D_i	C_h^i	C_s^i	S_p^i	θ
1	40	8	0.6	0.8	0.02	6.25	200	0.07	3	5	0.03
2	35	8.5	0.5	1.2	0.015	7.50	300	0.08	3.5	6	0.03

Solving the problem numerically with the help of computer software **MATHEMATICA 8.0**, we find that the optimum solution is P_1^*=204.9272, P_2^*=305.9327, ETP= 359.534

9.5 Sensitivity Analysis and Observations:

Table 9.5.1: Variation in the cost of ideal time (W_i):

Change in %	P_1^*	P_2^*	ETP
-50%	-0.264	0.0	-3.801
-25%	-0.131	0.0	-2.908
+25%	+0.131	0.0	+1.895
+50%	+0.260	0.0	+3.785

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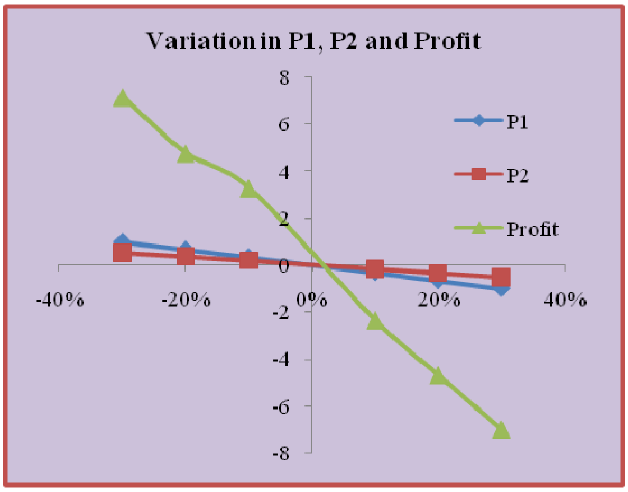

Fig. 9.3: Graphical representation of sensitivity of the P_1, P_2 and Profit w.r.t W_1&W_2

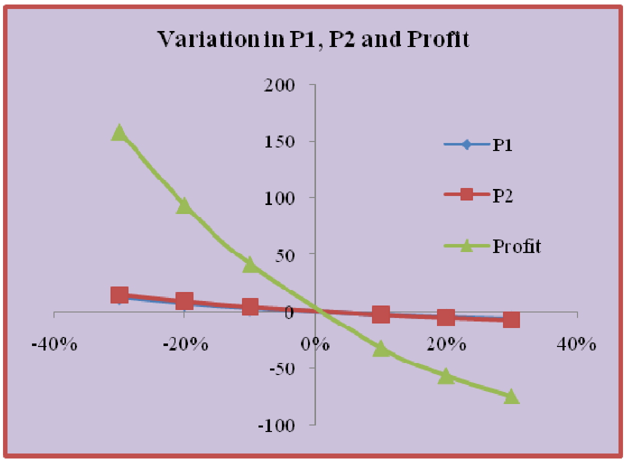

Fig. 9.4: Graphical representation of sensitivity of the P_1, P_2 and Profit w.r.t μ_1& μ_2

Table 9.5.2: Variation in the various parameters:

Parameter	% Change	-30%	-20%	-10%	10%	20%	30%
W_1&W_2	P_1	+0.96	+0.64	+0.32	-0.33	-0.67	-1.01
	P_2	+0.52	+0.35	+0.18	-0.18	-0.36	-0.54
	ETP	+7.16	+4.76	+3.27	-2.36	-4.70	-7.02
μ_1& μ_2	P_1	+13.02	+7.48	+3.29	-2.66	-4.86	-6.7
	P_2	+14.62	+8.53	+3.79	-3.10	-5.69	-7.89
	ETP	+158.09	+93.6	+41.45	-32.27	-56.77	-74.78
m_1&m_2	P_1	-1.61	-0.92	-0.40	+0.32	+0.59	+0.81
	P_2	-0.84	-0.49	-0.21	+0.17	+0.32	+0.44
	ETP	-12.01	-7.05	-3.15	+2.59	+4.76	+6.61
S_p&S_{2p}	P1	+26.2	+16.2	+6.0	-6.0	-16.2	-26.2
	P2	+40.28	+29.45	+16.08	-16.08	-29.45	-40.28
	ETP	+42.24	+38.35	+32.04	-32.04	-38.35	-42.24

9.10 Conclusion:

For the first time EMQ model is developed in presence of fuzzy environment under random deterioration. In a competitive market, price of goods plays an important factor. Generally, a reduced price encourages a customer to buy more. For fitting in with realistic circumstances, the environment of the whole study has been taken as fuzzified. Model is developed in both the environment crisp and imprecise. The reason for adaptation of this model is-(1) the execution of fuzzy random variables as demand and production gives more realistic information where the variable values are indefinite. (2) Incorporation of imprecision and improbability in machine breakdown production process. (3) Capacity constraint is also a realistic situation. From the analysis of the crisp model it has been observed that (a) The cost of ideal time of management units is indirectly proportional to the production rate and the profit. (b) Mean time of successive breakdowns is reversely proportional to the production rate and the profit. (c) The mean time to repair gives the reverse effect on the production rate and the profit. (d) The backlogging rate is indirectly proportional to the production rate and the profit.

We have many real life situations in which multi items inventories are required. For instance: a pharmacist keeps a number of medicines of different brands, readymade clothes shop keeps dresses of different things in different colors and in different size, shoe store stocks shoes of various models and sizes. The presented model is much more realistic and practical. The present model can be extended to include the delay in payments, inflation, and stochastic demand.

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